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by

D. A. Marshall



los alamos
scientific laboratory
of the University of California
LOS ALAMOS, NEW MEXICO 87544



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# NEUTRON ENERGY AND VELOCITY DISTRIBUTIONS FROM THERMAL D-T REACTIONS



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ABSTRACT

This report collects a number of general formulae which apply to the determination of the neutron energy and velocity distributions from the thermal D-T reaction. Comparison of the predictions of these formulae with the results of a Monte Carlo calculation suggests that the formulae apply to temperatures in excess of 100 keV.

## I. INTRODUCTION

Section II presents the general formulae applying to the thermal D-T → nα reaction. Section III applies these formulae and compares their predictions to a Monte Carlo calculation of the energy and velocity distributions.

## II. GENERAL FORMULAE

For reaction 1

$$1 + 3 \rightarrow 3 + 4 \quad ; \tag{1}$$

the relativistic energy of particle 4 in the center of momentum (mass) frame of 1 and 2 is given by[1]

$$E_{4c} = \left(W^2 - m_3^2c^4 + m_4^2c^4\right) \Big/ (2W) \quad , \tag{2}$$

where

$$W \equiv m_1c^2 + m_2c^2 + q \quad . \tag{3}$$

The rest energies of 1, 2, 3, 4 are $m_1c^2$, $m_2c^2$, $m_3c^2$, $m_4c^2$; q is any additional energy available in the center of momentum (mass) frame of particles 1 and 2; and c is the speed of light. The kinetic energy of 4 is given by

$$KE_4 = E_4 - m_4c^2 \quad . \tag{4}$$

The momentum is given by

$$P_4 = \sqrt{E_4^2 - m_4^2c^4}\,/c \tag{5}$$

$$= \sqrt{KE_4\left(2m_4c^2 + KE_4\right)}\,/c \quad . \tag{5´}$$

The velocity of 4 is given by

$$v_4 = P_4c^2/E_4 \quad . \tag{6}$$

If 1 and 2 have charge numbers $Z_1$ and $Z_2$, and if 1 and 2 are in a thermal distribution characterized by a temperature Θ (MeV), q is given by the distribution function which for Θ ≪ 1 MeV is approximately[2,3]

$$S(q) \simeq \exp\left[-B(q - q_0)^2\right] \quad , \tag{7}$$

where

$$q_0 = (1/2)^{2/3} E_g^{1/3} \Theta^{2/3} \quad , \tag{8}$$

$$B = 3\Big/\left(2^{4/3} E_g^{1/3} \Theta^{5/3}\right) \quad . \tag{9}$$

$E_g$ is the "Gamow energy" which is defined by

$$E_g = 2M\left(\pi Z_1 Z_2 e^2/\hbar\right)^2 \quad , \tag{10}$$

where M is the reduced mass of 1 and 2. Evaluating this expression,

$$E_g = 0.979\ Y\ (\text{MeV}) \quad , \tag{11}$$

where

$$Y = AZ_1^2Z_2^2 \tag{12}$$

and

$$A = A_1A_2/(A_1 + A_2) \quad . \tag{13}$$

Here $A_1$ and $A_2$ are the masses of 1 and 2 in amu.

The expectation value of q is approximately given by

$$\bar{q} = q^0 \tag{14}$$

$$= E_g^{1/3}\,\Theta^{2/3}/2^{2/3} \tag{15}$$

and the $\Delta q$ about $\bar{q}$ is

$$\Delta q = \sqrt{\left\langle q^2 - \bar{q}^2 \right\rangle} \quad , \tag{16}$$

$$= \sqrt{\frac{1}{2B}} \quad , \tag{17}$$

$$= \frac{1}{\sqrt{3}}\left(2E_G\right)^{1/6}\Theta^{5/6} \quad . \tag{18}$$

The energy of the center of momentum (mass) frame for 1 and 2 in the laboratory has distribution characterized by the temperature Θ. For the decay of this system into particles 3 and 4 where $KE_{4_0} \gg \Theta$ and for $KE_4 \simeq KE_{4_0}$, the distribution function is

$$S(KE_4) \simeq \exp\left(-\frac{A_1 + A_2}{A_4}\,\frac{KE_4 + KE_{4_0}}{\Theta}\right)$$

$$\times \sinh\left(2\,\frac{A_1 + A_2}{A_4}\,\frac{\sqrt{KE_4 KE_{4_0}}}{\Theta}\right) \tag{19}$$

where $KE_{4_0}$ is the kinetic energy of particle 4 in the center of momentum (mass) frame.

The expectation value of $KE_4$ is given by

$$\overline{KE_4} = KE_{4_0} + 3/2\,A_4/(A_1 + A_2)\,\Theta \tag{20}$$

and the $\Delta KE_4$ about $\overline{KE_4}$ is

$$\Delta KE_4 = \left[2\,\frac{A_4}{A_1 + A_2}\,\Theta\,KE_{4_0} + 3/2\left(\frac{A_4}{A_1 + A_2}\right)^2\Theta^2\right]^{1/2} \quad , \tag{21}$$

or

$$\Delta KE_4 \simeq \sqrt{2\,\frac{A_4}{A_1 + A_2}\,\Theta\,KE_{4_0}} \tag{21'}$$

for

$$KE_0 \gg \frac{A_4}{A_1 + A_2}\,\Theta \quad .$$

## III. NUMERICAL APPLICATIONS TO THE THERMAL DT→ nα REACTION

The following list gives the atomic masses on a carbon 12 scale.[4]

| | |
|---|---|
| $m_e$ | 0.000 548 579 |
| n | 1.008 665 22 |
| D | 2.014 102 22 |
| T | 3.016 049 72 |
| $^4$He | 4.002 603 26 |

The nuclear masses are obtained by correcting for the electron masses and electron binding energies which are 13.595 eV for H and 54.403 eV for He.[5]

| | |
|---|---|
| n | 1.008 665 22 |
| D | 2.013 553 66 |
| T | 3.015 501 16 |
| α | 4.001 506 16 |

The conversion from amu to MeV is 1 amu = 931.504 MeV. The speed of light used in the calculations is 299.792 50 mm/ns.

Substituting into Eq. (15), the expectation value of q is found to be

$$\bar{q} = 0.6647\ \Theta^{2/3}\ (\text{MeV}) \tag{22}$$

where Θ is in MeV. The associated width is found from Eq. (18) to be

$$\Delta q = 0.6657\ \Theta^{5/6}\ (\text{MeV}) \quad . \tag{23}$$

Propagating $\Delta q$ through Eq. (2), the resulting spread in the center of momentum frame energy is

$$\Delta KE_{4c} = (1 - E_{4c}/W)\Delta q \tag{24}$$

$$\simeq 4/5\ \Delta q \tag{25}$$

$$\simeq 0.078 \text{ MeV at 100 keV.} \tag{26}$$

On the other hand, $\Delta KE_4$ from Eq. (21) is

$$\Delta KE \simeq \sqrt{(2/5)14\Theta} \tag{27}$$

$$\simeq 0.748 \text{ MeV at 100 keV.} \tag{28}$$

Comparing Eqs. (26) and (28), it is clear that for temperatures below 100 keV the effect of the spread of the energies caused by $\Delta q$ can be ignored in comparison to the spread of energies caused by $\Delta KE_4$ from Eq. (21).

The velocity of the neutrons is determined by the following scheme. The $\bar{q}$ is calculated from Eq. (22). $E_{4c}$ is calculated from Eq. (2). The kinetic energy, $KE_{40}$, is calculated from Eq. (4). The kinetic energy distribution in the laboratory, $KE_4$, is obtained from Eq. (19) which may be written as

$$S(KE_4) \simeq \exp\left[-\frac{A_1 + A_2}{A_4}\frac{1}{\Theta}\left(\sqrt{KE_4} - \sqrt{KE_{40}}\right)^2\right] \quad . \tag{29}$$

The expected kinetic energy in the laboratory is given by Eq. (20). The neutron velocity at the expected kinetic energy is obtained from Eqs. (5) and (6). The velocity distribution may be obtained from Eq. (29) through the use of Eqs. (4), (5), and (6). Near the peak, Eq. (29) may be approximated by

$$S(KE_4) \simeq \exp\left[-(A_1 + A_2)/(4A_4)(KE_4 - KE_{40})^2/(KE_{40}\Theta)\right] \quad . \tag{30}$$

Table I gives the expected neutron kinetic energy as a function of $\Theta$, the expected $\Delta KE$, the expected velocity, and the $\Delta v$ for the $D\text{-}T \rightarrow n\alpha$ reaction.

TABLE I

ENERGY AND VELOCITY VS TEMPERATURE

| $\Theta$ (keV) | $\overline{KE_4}$ (MeV) | $\Delta KE_4$ (MeV) | v (mm/ns) | $\Delta v$ (mm/ns) |
|---|---|---|---|---|
| 0 | 14.0290 | 0.000 | 51.2344 | 0.000 |
| 0.1 | 14.030 | 0.024 | 51.236 | 0.042 |
| 0.2 | 14.031 | 0.034 | 51.238 | 0.060 |
| 0.3 | 14.031 | 0.041 | 51.239 | 0.073 |
| 0.4 | 14.032 | 0.047 | 51.240 | 0.085 |
| 0.5 | 14.032 | 0.053 | 51.241 | 0.095 |
| 0.6 | 14.033 | 0.058 | 51.24 | 0.10 |
| 0.7 | 14.033 | 0.063 | 51.24 | 0.11 |
| 0.8 | 14.034 | 0.067 | 51.24 | 0.12 |
| 0.9 | 14.034 | 0.071 | 51.24 | 0.13 |
| 1.0 | 14.035 | 0.075 | 51.24 | 0.13 |
| 2.0 | 14.04 | 0.11 | 51.25 | 0.19 |
| 3.0 | 14.04 | 0.13 | 51.26 | 0.23 |
| 4.0 | 14.04 | 0.15 | 51.26 | 0.27 |
| 5.0 | 14.05 | 0.17 | 51.26 | 0.30 |
| 6.0 | 14.05 | 0.18 | 51.27 | 0.33 |
| 7.0 | 14.05 | 0.20 | 51.27 | 0.35 |
| 8.0 | 14.05 | 0.21 | 51.28 | 0.38 |
| 9.0 | 14.05 | 0.22 | 51.28 | 0.40 |
| 10.0 | 14.06 | 0.24 | 51.28 | 0.42 |
| 20.0 | 14.07 | 0.34 | 51.31 | 0.60 |
| 30.0 | 14.09 | 0.41 | 51.34 | 0.73 |
| 40.0 | 14.10 | 0.48 | 51.37 | 0.85 |
| 50.0 | 14.12 | 0.53 | 51.39 | 0.95 |
| 60.0 | 14.13 | 0.58 | 51.41 | 1.0 |
| 70.0 | 14.14 | 0.63 | 51.43 | 1.1 |
| 80.0 | 14.15 | 0.67 | 51.45 | 1.2 |
| 90.0 | 14.16 | 0.72 | 51.47 | 1.3 |
| 100.0 | 14.17 | 0.75 | 51.49 | 1.3 |

A Monte Carlo code was written which sampled the D and T from a Maxwellian distribution, evaluated the cross section to obtain the reaction probability and produced the neutron isotropically in the center of momentum frame of the D and T, and translated the neutron back to the laboratory frame in a relativistically correct manner. Table II compares the calculated Monte Carlo results with the numbers from Table I.

At 1 and 10 keV, the kinetic energy distributions were compared to the distribution predicted by Eq. (29) where $KE_{40}$ was the mean calculated by

TABLE II

ENERGY AND VELOCITY COMPARED TO THE RESULTS OF A MONTE CARLO INTEGRATION

| | Table I | | MC Code | | Table I | | MC Code | |
|---|---|---|---|---|---|---|---|---|
| Θ (keV) | KE (MeV) | ΔKE (MeV) | KE (MeV) | ΔKE (MeV) | v (mm/ns) | Δv (mm/ns) | v (mm/ns) | Δv (mm/ns) |
| 1 | 14.035 | 0.075 | 14.036 | 0.077 | 51.24 | 0.13 | 51.25 | 0.14 |
| 10 | 14.06 | 0.24 | 14.06 | 0.24 | 51.28 | 0.42 | 51.30 | 0.43 |
| 100 | 14.17 | 0.75 | 14.14 | 0.77 | 51.5 | 1.3 | 51.4 | 1.4 |

the Monte Carlo code. To within the statistics for the calculation, the two distributions agreed.

Table II suggests that the formulae presented here are valid for temperatures in excess of 100 keV.

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